18. Use Stirling's formula to find $f(1.22)$ from the following table.

x :	1.0	1.1	1.2	1.3	1.4
$f(x)$:	0.84147	0.89121	0.93204	0.96356	0.98545
x :	1.5	1.6	1.7	1.8	
$f(x)$:	0.99749	0.99957	0.99385	0.97385	

19. Ten competitors in a beauty contest are ranked by three judges in the following order

Judge I :	1	6	5	10	3	2	4	9	7	8
Judge II :	3	5	8	4	7	10	2	1	6	9
Judge III :	6	4	9	8	1	2	3	10	5	7

Use rank correlation coefficient to determine which pair of Judges has the nearest approach to common tastes in beauty.

20. Find the standard deviation of the binomial distribution.

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17PMAED1

(For the candidates admitted from 2017–2018 onwards)

P.G. DEGREE EXAMINATION, NOVEMBER 2018

Second Semester

NUMERICAL AND STATISTICAL METHODS

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is the order of convergence of Newton-Raphson method?
2. Explain bisection method.
3. Solve by Gauss elimination method, $11x + 3y = 17$, $2x + 7y = 16$.
4. What is iterative method?
5. What is interpolation?
6. Write Newton's forward formula for interpolation.
7. Write the property of correlation coefficient.
8. Write any two uses of regression analysis.
9. What is the mean of the binomial distribution?
10. Write any two properties of the normal distribution.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Solve for a positive root of $x^3 - 4x + 1 = 0$, using regula falsi method.

Or

(b) Use Newton-Raphson method to find the value of $\sqrt{5}$.

12. (a) Solve the following system of equations by Gauss elimination method.

$x + 2y + z = 3, 2x + 3y + 3z = 10, \; 3x - y + 2z = 13$

Or

(b) Solve the following system of equations by Gauss-Jordan method

$10x + y + z = 12, \; x + 10y + z = 12,$
$x + y + 10z = 12$

13. (a) Find the value of y when $x = 28$ from the following table.

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

Or

(b) Evaluate $\int_0^6 \frac{1}{1+x} dx$ by using

(i) Trapezoidal Rule.

(ii) Simpson's one-third Rule.

14. (a) Making use of the data summarized below, calculate the coefficient of correlation.

X_1:	10	6	9	10	12	13	11	9
X_2:	9	4	6	9	11	13	8	4

Or

(b) Find the regression equation of Y on X from the following table.

X:	25	28	35	32	31	36	29	38	34	32
Y:	43	46	49	41	36	32	31	30	33	39

15. (a) A coin is tossed six times. What is the probability of obtaining four or more heads.

Or

(b) The mean weight of 500 male students in a certain college is 151lb and the standard deviation is 15lb. Assuming the weights are normally distributed, find how many students weight between 120lb and 155lb.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the positive root of $x^3 - x - 1 = 0$ correct to 4 decimal places using bisection method.

17. Solve the following system of equations by Gauss-Seidel method.

$10x - 5y - 2z = 3, \; 4x - 10y + 3z = -3, x + 6y + 10z = -3$

S.No. 240

17PMAE06

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Third Semester

Mathematics

PROGRAMMING WITH C++

Time : Three hours — Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Expand : (a) OOP and (b) POP.

2. Define polymorphism

3. Define enumerated data type.

4. Define operator overloading.

5. How does the main function in C and C++ differ?

6. Write any two characteristics of static member variable.

7. Define constructor.

8. What is the difference between overloading of binary operators and unary operators?

9. What are input and output streams?

10. Define asynchronous exceptions.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) List some benefits of OOP technology.

Or

(b) Discuss the programming structures of C++ language.

12. (a) Write short note on reference variables.

Or

(b) Describe the switch statement in C++ with an example.

13. (a) Write short note on call by reference.

Or

(b) Explain friendly functions in C++.

14. (a) Explain briefly the parameterized constructors.

Or

(b) Write the rules of overloading operators.

15. (a) Draw the flow chart for stream classes for file operations.

Or

(b) Describe the role of keywords try, catch and throw in exception handling.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Explain the basic concepts of object oriented programming in detail.

17. Explain the tokens in C++ language.

18. Write short note on inline function with suitable example.

19. Write a C++ program to construct a matrix of size $m \times n$.

20. How to open and closing a file with suitable example?

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(6 pages)
S.No. 231

17PMAE03

(For the candidates admitted from 2017-2018 onwards)

M.Sc. DEGREE EXAMINATION,

NOVEMBER 2018.

Second Semester

Mathematics

DISCRETE MATHEMATICS

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Construct the truth table of the $p \wedge \neg q \to (p \wedge q)$.

2. Define Predicate.

3. State the Sum rule principle.

4. State the Pigeonhole principle.

12. (a) If n is a positive integer and r is an integer with $1 \le r \le n$, then prove that there are $P(n,r) = n(n-1)\cdot(n-2)\ldots(n-r+1)$, r – permutations of a set with n distinct elements.

Or

(b) Prove that the number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, ..., and n_k indistinguishable objects of type k, is $\frac{n!}{n_1!n_2!\ldots n_k!}$.

13. (a) A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let a_n be the number of valid n-digit codewords. Find the recurrence relation for a_n.

Or

(b) Find all solutions of the recurrence relation $a_n = 3a_{n-1} + 2n$. What is the solution with $a_1 = 3$.

14. (a) Show that the distributive law $x(y+z) = xy + xz$ is valid.

Or

(b) Construct circuits that produce the following outputs:

(i) $\overline{x(y+\overline{z})}$

(ii) $(x+y+z)(\overline{x}\,\overline{y}\,\overline{z})$.

15. (a) What are the Kleene closures of the sets $A = \{0\}$, $B = \{0,1\}$ and $C = \{1,1\}$?

Or

(b) Construct a Turing Machine for adding two nonnegative integers.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that

(a) $\neg(p \to q)$ and $p \wedge \neg q$

(b) $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$

are logically equivalent by developing a series of logical equivalence.

[P.T.O.]

17. How many Poker hands of five cards can be dealt from a standard deck of 52 Cards? Also how many ways are there to select 47 cards from a standard deck of 52 cards?

18. Use generating functions to find the number of ways to select r objects of n different kinds if we must select at least one object of each kind.

19. Use K-maps to minimize these sum-of-product expansions

(a) $xy\bar{z} + x\bar{y}\,\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}\,\bar{z}$

(b) $x\bar{y}z + x\bar{y}\,\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}z + \bar{x}\,\bar{y}\,\bar{z}$

(c) $xy\bar{z} + x\bar{y}\,\bar{z} + \bar{x}\,\bar{y}z + \bar{x}\,\bar{y}\,\bar{z}$.

20. Construct deterministic finite-state automata that recognize each of these languages.

(a) The set of bit strings that begin with two 0's

(b) The set of bit strings that contain two consecutive 0's

(c) The set of bit strings that do not contains two consecutive 0's

(d) The set of bit strings that end with two 0's.

The set of bit strings that contain at least two 0's.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve $\frac{dy}{dx} = x + z$, $\frac{dy}{dx} = x - y^2$ with $y(0) = 2$, $z = (0) = 1$ to get $y(0.1), y(0.2), z(0.1)$ and $z(0.2)$ approximately by Taylor's algorithm.

17. Approximate y and z at $x = 0.1$ using Picard's method for the solution to the equation $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y + z)$, given that $y(0) = 1$ and $z(0) = \frac{1}{2}$.

18. Using Runge-Kutta methods of fourth order, solve for $y(0.1), y(0.2)$ and $y(0.3)$ given that $y' = xy + y^2$, $y(0) = 1$.

19. Solve $u_{xx} + u_{yy} = 0$ in $0 \le x \le 4$, $0 \le y \le 4$, given that $u(0, y) = 0$, $u(4, y) = 81.2\, y$; $u(x, 0) = \frac{x^2}{2}$ and $u(x, 4) = x^2$. Take $h = k = 1$ and obtain the result correct to one decimal, using Liebmann's iteration process.

20. Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary and mesh length = 1.

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7. Define diagonal five point formula.

8. Define gird points.

9. Write the formula for Crank-Nicholson scheme.

10. How will you classify partial differential equation of second order?

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Using Taylor's series method, solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$ and 0.4.

Or

(b) Using Adams-Bashforth method, find $y(1.4)$ given $y' = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$.

12. (a) Use Picard's method to approximate the value of y when $x = 0.1, 0.2, 0.3$ given that $y = 1$ when $x = 0$ and $y = 1 + xy$.

Or

(b) Solve $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ in the range $0 \le x \le 0.3$ using Euler's method.

13. (a) Apply Runge's method to find an approximate value of y when $x = 0.2$ given that $y' = x + y$ and $y(0) = 1$.

Or

(b) Given $y' = x^2 - y$, $y(0) = 1$ find $y(0.1)$, $y(0.2)$ using Runge-Kutta method of second order.

14. (a) Classify the following equations.

(i) $u_{xx} - 6u_{xy} + 9u_{yy} - 17u_y = 0$

(ii) $3u_{xx} + u_{xy} - 4u_{yy} + 3u_y = 0$.

Or

(b) State Standard Five Point Formula (SFPF) and Diagonal Five Point Formula (DFPF).

15. (a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ inside a square region bounded by the lines $x = 0$, $x = 4$, $y = 0$, $y = 4$ given that $u = x^2y^2$ at the boundary, using relaxation technique.

Or

(b) Solve $u_{tt} = 4u_{xx}$ with the boundary conditions $u(0,t) = 0 = u(4,t)$, $u_t(x,0)$ and $u(x,0) = x(4-x)$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and Prove Brachistochrone problem.

17. Using only the basic necessary condition $\delta I = 0$, find the curve on which an extremum of the functional $I[y(x)] = \int_0^{x_1} \frac{(1+y'^2)^{1/2}}{y}\, dx,\ y(0) = 0$ can be achieved if the second boundary point (x_1, y_1) can move along the circumference $(x-9)^2 + y^2 = 9$.

18. Reduce the following boundary value problem into an integral equation: $\frac{d^2y}{dx^2} + \lambda y = 0$ with $y(0) = y(1) = 0$.

19. Derive the solution of Fredholm integral equation of the second kind using Separable kernel method.

20. State and prove Hilbert — Schmidt theorem.

S.No. 238

17PMA11

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Third Semester

Mathematics

CALCULUS OF VARIATION AND INTEGRAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State Hertz'z principle.
2. Write the Euler-Poisson equation.
3. Define transversality condition.
4. State Fermat's principle.
5. Define Eigen value and Eigen function.
6. Define resolvent kernel.
7. Write the separable kernel of Fredholm integral equation.

8. Write the Volterra integral equation of the second kind.

9. Define complex Hilbert space.

10. When we say the orthonormal system of function is normalized.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If for every continuous function $\eta(x), \int_a^b \phi(x)\eta(x)dx = 0$ where $\phi(x)$ is continuous in the closed interval $[a, b]$ then show that $\phi(x) = 0$ on $[a, b]$.

Or

(b) Find the Euler-Ostrogradsky equation for $I[u(x, y)] = \iint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] dx\, dy$ where the values of u are prescribed on the boundary Γ of the domain D.

12. (a) Find the extremals with corner points of the functional

$$I[y(x)] = \int_{x_1}^{x_2} y'^2(1 - y')^2 dx.$$

Or

(b) Find the shortest path from the point $A(-2, 3)$ to the point $B(2, 3)$ located in the region $y \leq x^2$.

13. (a) Show that the function $y(x) = (1 + x^2)^{-3/2}$ is a solution of the Volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt.$$

Or

(b) Convert the differential equation $y'' + \lambda xy = f(x), y(0) = 1, y'(0) = 0$.

14. (a) Solve : $y(x) = f(x) + \lambda \int_0^1 xt\, y(t)dt$.

Or

(b) Find the resolvent kernel of $y(x) = f(x) + \lambda \int_0^x e^{x-t} y(t)dt$.

15. (a) Find the eigen value of the symmetric integral equation $y(x) = (x + 1)^2 + \int_{-1}^{1} (xt + x^2t^2) y(t)dt$ by using Hilbert-Schmidt theorem.

Or

(b) Explain Gram-Schmidt orthogonolization method.

(b) For each real number α the set $\{x: f(x) \geq \alpha\}$ is measurable.

(c) For each real number α the set $\{x: f(x) < \alpha\}$ is measurable.

(d) For each real number α the set $\{x: f(x) \leq \alpha\}$ is measurable. These statements imply

(e) For each extended real number α the set $\{x: f(x) = \alpha\}$ is measurable.

17. Let f be defined and bounded on a measurable set E with mE finite. In order that $\inf_{f \leq \psi} \int_E \psi(x)\, dx = \sup_{f \geq \varphi} \int_E \varphi(x)\, dx$ for all simple functions φ and ψ. Prove that it is necessary and sufficient that f be measurable.

18. If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ almost everywhere then prove that f is constant.

19. If $E_i \, \varepsilon \, B$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu E_n$.

20. Prove that the class $\mathcal{B}$ of μ^* - measurable sets is a σ-algebra. If $\bar{\mu}$ is μ^* restricted to $\mathcal{B}$, then $\bar{\mu}$ is a complete measure on $\mathcal{B}$.

(For the candidates admitted from 2017-2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Third Semester

Mathematics

MEASURE THEORY AND INTEGRATION

Time : Three hours   Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define countable subadditivity.

2. Define outer measure.

3. Define step function.

4. Define simple function.

5. Define the set of four derivatives.

6. Define absolutely continuous.

7. Define mutually singular.

8. Define saturated.

9. Define regular.

10. Define semi algebra.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let A be any set, and $E_1, \ldots, E_n$ a finite sequence of disjoint measurable set. Then prove that $m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$.

Or

(b) Let E be a measurable set of finite measure, and $\langle f_n \rangle$ a sequence of measurable functions defined on E. Let f be a real-valued function such that for each x in E we have $f_n(x) \to f(x)$. Then prove that given $\in > 0$ and $\delta > 0$, there is a measurable set $A \subset E$ with $mA < \delta$ and an integer N such that for all $x \notin A$ and all $n \geq N$, $|f_n(x) - f(x)| < \in$.

12. (a) State and prove bounded convergence theorem.

Or

(b) State and prove monotone convergence theorem.

13. (a) Show that a function f of bounded variation on $[a, b]$ if and only if f is the difference of two monotone real-valued functions on $[a, b]$.

Or

(b) Show that if f is absolutely continuous on $[a, b]$, then it is of bounded variation on $[a, b]$.

14. (a) If f and g are nonnegative measurable functions and a and b nonnegative constants, then prove $\int af + bg = a\int f + b\int g$.

Or

(b) State and prove Lebesgue decomposition theorem.

15. (a) Show that the set function μ^* is an outer measure.

Or

(b) State and prove Fubini theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let f be an extended real-valued function whose domain is measurable. Then show that the following statements are equivalent:

(a) For each real number α the set $\{x : f(x) > \alpha\}$ is measurable.

17. State and prove the uniform limit theorem.

18. Prove that if L is a linear continuum in the order topology, then L is connected, and so are the intervals and rays in L.

19. State and prove the Lebesgue number theorem.

20. State and prove Urysohnmetrization theorem.

S.No. 236

17PMA09

(For the candidates admitted from 2017-2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Third Semester

Mathematics

TOPOLOGY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define topological space.

2. Define limit point.

3. Define homeomorphism.

4. Define product topology.

5. Define linear continuum.

6. Define components.

7. Define isolated point.

8. Define limit point compact.

9. Define regular.

10. Define completely normal.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) If X is a set and if $\mathfrak{B}$ is a basis for a topology $\mathcal{T}$ on X. Then prove that $\mathcal{T}$ equals the collection of all unions of elements of $\mathfrak{B}$.

Or

(b) Prove that every finite point set in a Hausdroff space is closed.

12. (a) State the rules for constructing continuous functions. Let d and d' be two metrics on the set X, let $\mathcal{T}$ and $\mathcal{T}'$ be the topologies they induce, respectively. Show that $\mathcal{T}'$ is finer than $\mathcal{T}$ if and only if for each x in X and each $\in > 0$, there exists a $\delta > 0$ such that $B_{d'}(x,\delta) \subset B_d(x,\in)$.

Or

(b) State and prove the sequence lemma.

13. (a) State and prove intermediate value theorem.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

14. (a) Prove that every closed subspace of a compact space is compact.

Or

(b) State and prove the extreme value theorem.

15. (a) Let X be a topological space. Let one-point sets in X be closed then prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Show that every compact Hausdorff space is normal.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then show that $\overline{A} = A \cup A'$.

Answer ALL questions.

1. Define irreducible operator.

2. Define adjoint operator.

3. Define mixed boundary value problem.

4. Write Laplace equation in cylindrical coordinates.

5. State Robin's boundary condition.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If $u = f(x+iy) + g(x-iy)$, where f and g arbitrary functions, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Or

(b) Construct an operator adjoint to the Laplace operator given by $L(u) = u_{xx} + u_{yy}$.

13. (a) The ends A and B of a rod, 10 cm in length are kept at temperature 0°C and 100°C respectively until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C. Find the temperature distribution in rod at time t.

Or

(b) Derive the diffusion equation in cylindrical coordinates and thus find its general solution.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that for the equation $\frac{\partial^2 z}{\partial x \partial y} + \frac{1}{4} z = 0$ the Green's function is given by $w(x, y, \xi, \eta) = J_0\left\{\sqrt{(x-\xi)(y-\eta)}\right\}$ where $J_0(z)$ is the Bessel function of first kind and of order zero.

17. Derive the solution for interior Dirichlet problem for a circle.

18. If $\theta(r, t)$ satisfies the equation

(a) $\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = \frac{1}{k}\frac{\partial \theta}{\partial t}$, $0 \le r \le a$, $t > 0$.

(b) $\theta(r, 0) = f(a)$, $0 \le r \le a$

(c) $\frac{\partial \theta}{\partial r} + h\theta$, at $r = a$, $\forall t > 0$.

Then show that

$$\theta(r, t) = \frac{2}{a^2} \sum_i \frac{\xi_1^2 e^{-k\xi_1^2 t} J_0(\xi_i r)}{(h^2 + \xi_i^2)[J_0(\xi_i a)]^2} \int_0^a u\, f(u) J_0(\xi_i u) du$$

where the sum is taken over the positive roots $\xi_1, \xi_2, \xi_3, \ldots \xi_i, \ldots$ of the equation $hJ_0(a\xi_i) = \xi_i J(a\xi_i)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $f(z)$ is analytic in a region Ω, containing a then prove that it is possible to write $f(z)=f(a)+\frac{f'(a)}{1!}(z-a)+\frac{f''(a)}{2!}(z-a)^2+....+\frac{f^{(n-1)}(a)}{(n-1)!}(z-a)^{n-1}+f_n(z)(z-a)^n$.

Where $f_n(z)$ is analytic in Ω.

17. State and prove argument principle.

18. Prove that the function $P_U(z)$ is harmonic for $|z|<1$ and $\lim_{Z\to e^{i\theta_0}} P_U(z)=U(\theta_0)$ provided that U is continuous at θ_0.

19. Prove that a family of analytic or meromorphic functions f is normal in the classical sense it and only if the expressions $\rho(f)=\frac{2|f'(z)|}{1+|f(z)|^2}$ are locally bounded.

20. State and prove Schwarz-Christoffel formula.

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17PMA07

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Second Semester

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. If $f(z)$ is analytic in an open disk Δ, then prove that $\int_\gamma f(z)\,dz=0$ for every closed curve γ in Δ.
2. State Liouville's theorem.
3. Define simply connected region.
4. State Rouche' s theorem.
5. Define Poisson integral.
6. State Hurwitz theorem.
7. Write down the Jensen's formula.
8. Define normal.
9. Define free boundary arc.
10. Define elliptic integral.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Suppose that $f(z)$ is analytic in an open disk Δ, and let γ be a closed curve in Δ. Prove that for any point a not on γ $n(\gamma, a) \cdot f(a) = \frac{1}{2\pi i} \int_\gamma \frac{f(z)\,dz}{z-a}$ where $n(\gamma, a)$ is the index of a with respect to γ.

Or

(b) If $f(z)$ is analytic for $|z|<1$ and satisfies the conditions $|f(z)| \le 1$, $f(0)=0$ then prove that $|f(z) \le |z||$ and $|f'(0)| \le 1$. Also if $|f(z)| = |z|$ for some $z \ne 0$, or if $|f'(0)| = 1$, then prove that $f(z) = cz$ with a constant c has absolute value 1.

12. (a) Let $f(z)$ be analytic expect for isolated singularities a_j in a region Ω. Prove that $\frac{1}{2\pi i} \int_\gamma f(z)\,dz = \sum_j n(\gamma, a_j) \operatorname{Res}_{z=a_j} f(z)$ for any cycle γ which is homologous to zero in Ω and does not pass through any of the points a_j.

Or

(b) Evaluate $\int_0^\pi \log \sin\theta \, d\theta$.

13. (a) Derive $u(a) = \frac{1}{2\pi} \int_{|Z|=R} \frac{R^2 - |a|^2}{|Z-a|^2} u(z)\,d\theta \frac{1}{2\pi}$

$\int_{|Z|=R} \operatorname{Re} \frac{z+a}{z-a} u(z)\,d\theta$.

Or

(b) Discuss the reflection principle.

14. (a) Prove that a family $\mathfrak{F}$ of analytic functions is normal with respect to C if and only if the functions in $\mathfrak{F}$ are uniformly bounded on every compact set.

Or

(b) Prove that a family $\mathfrak{F}$ is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\in > 0$ it is possible to find $f_1, \ldots, f_n \in \mathfrak{F}$ such that every $f \in \mathfrak{F}$ satisfies $d(f, f_j) < \in$ on E for some f_j.

15. (a) Prove that a continuous function $u(z)$ which satisfies $u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + r e^{i\theta})\,d\theta$ is, necessarily harmonic.

Or

(b) Derive Harnack's inequality.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove first proof Sylow's theorem.

17. Show that every finite abelian group is the direct product of cyclic groups.

18. State and prove first isomorphism theorem.

19. If K is a finite extension of F, then $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K : F]$.

20. State and prove Frobenius theorem.

17PMA05

(For the candidates admitted from 2017-2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

Second Semester

Mathematics

ALGEBRA

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define conjugate of a in G.

2. Define p-Sylow subgroup of G.

3. Define invariants of G.

4. Define normal subgroup.

5. Define right regular module.

6. State Zorn's lemma.

7. When a group is said to be solvable.

8. Define splitting field.

9. State Jacobson theorem.

10. Define finite field.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If A, B are finite subgroups of G then show that $o(A x B) = \frac{o(A)\, o(B)}{o(A \cap xBx^{-1})}$.

Or

(b) Show that conjugacy is an equivalence relation in group G.

12. (a) f G and G' are isomorphic abelian group then show that for every integer s, $G(s)$ and $G'(s)$ are isomorphic.

Or

(b) If G is the internal direct product of $N_1, \ldots, N_n$. Then for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i$, $b \in N_j$ then prove that $ab = ba$.

13. (a) Prove that every cyclic module is isomorphic to a quotient module of the regular module by some right ideal.

Or

(b) State and prove Modular law.

14. (a) Show that the fixed field of G is a subfield of K.

Or

(b) Let $G = S_n$, where $n \geq 5$; then prove that $G^{(K)}$ for $K = 1, 2, \ldots$, contains every 3-cycle of S_n.

15. (a) If the finite field F has p^m elements then show that the polynomial $x^{p^m} - x$ in $F[x]$ factors in $F[x]$ as $x^{p^m} - x = \prod_{\lambda \in F} (x - \lambda)$.

Or

(b) Let C be the field of complex numbers and suppose that the division ring D is algebraic over C. Then prove that $D = C$.

(b) Let A be the complex 3×3 matrix $\begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$ then find its minimal polynomial.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let V and W be finite-dimensional vector spaces over the field F such that $\dim V = \dim W$. If T is a linear transformation from V into W, then show that the following are equivalent :

    (a) T is invertible.

    (b) T is non-singular.

    (c) T is onto, (i.e.) the range of T is W.

17. Let F be a field and a be a linear algebra with identity over F. Suppose f and g are polynomials over F, that α is an element of a and that c belongs to F. Then show that

    (a) $(cf + g)(\alpha) = cf(\alpha) + g(\alpha)$ and

    (b) $(fg)(\alpha) = f(\alpha)\, g(\alpha)$.

18. State and prove Cayley-Hamilton theorem.

19. State and prove primary decomposition theorem.

20. State and prove cyclic decomposition theorem.

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17PMA01

(For the candidates admitted from 2017-2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2018.

First Semester

Mathematics

LINEAR ALGEBRA

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define nullity.

2. Define annihilator.

3. Define prime polynomial over the field F.

4. Define ring.

5. Define diagonalizable.

6. Define classical adjoint of matrix.

7. Define skew-symmetric matrix.

8. Define nilpotent.

9. State generalized Cayley-Hamilton theorem.

10. Define Jordan matrix.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite-dimensional. Then show that $Rank(T) + Nullity(T) = \dim V$.

Or

(b) If S is any subset of a finite-dimensional vector space then show that $(S^0)^0$ is the subspace spanned by S.

12. (a) Suppose f and d are non-zero polynomials over a field F such that $\deg d \leq \deg f$. Then show that there exists a polynomial g in $F[x]$ such that either $f - dg = 0$ or $def(f - dg) < \deg f$.

Or

(b) Prove that a linear combination of n-linear function is n-linear.

13. (a) Let K be a commutative ring with identity and let A and B be $n \times n$ matrices over K. Then prove that det $(AB) = \det(A)\det(B)$.

Or

(b) Let A be the real 3×3 matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ then find the characteristic value of A.

14. (a) Let W be an invariant subspace for T. The characteristic polynomial for the restriction operator T_W divides the characteristic polynomial for T. Then show that the minimal polynomial for T_W divides the minimal polynomial for T.

Or

(b) Let $\Im$ be a commuting family of diagonalizable linear operators on the finite-dimensional vector space V. Show that there exists an ordered basis for V such that every operator in $\Im$ is represented in that basis by a diagonal matrix.

15. (a) Let T be a linear operator on a finite-dimensional vector space V. Then show that there exists a vector α in V such that the T-annihilator of α is the minimal polynomial for T.

Or

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7. Define two dimensional flow.

8. Define complex velocity potential.

9. Define the stress matrix.

10. Write the Navier-Stokes equation in Tensor and Vector form.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Discuss the local and particle rates of change.

Or

(b) Derive the equation of continuity.

12. (a) Explain: The pitot tube.

Or

(b) Derive Euler's equation of motion.

13. (a) If $r^n S_n(\theta, \Psi)$ is a harmonic function then prove that $r^{-(n+1)} S_n(\theta, \Psi)$ is also harmonic.

Or

(b) Discuss an image of a source in a solid sphere.

14. (a) Discuss the flow for which $w = z^2$.

Or

(b) State and prove Mime-Thomson circle theorem.

15. (a) Discuss translational motion of fluid element.

Or

(b) Express the six distinct components of the stress matrix in terms of the principal stresses.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. At the point in an incompressible fluid having spherical polar co-ordinates (r, θ, Ψ), the velocity components are $[2Mr^{-3} \cos\theta, Mr^{-3} \sin\theta, 0]$, where M is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of the streamlines.

17. Prove that at any point P of a moving inviscid fluid the pressure p is the same in all directions.